APPENDIX I

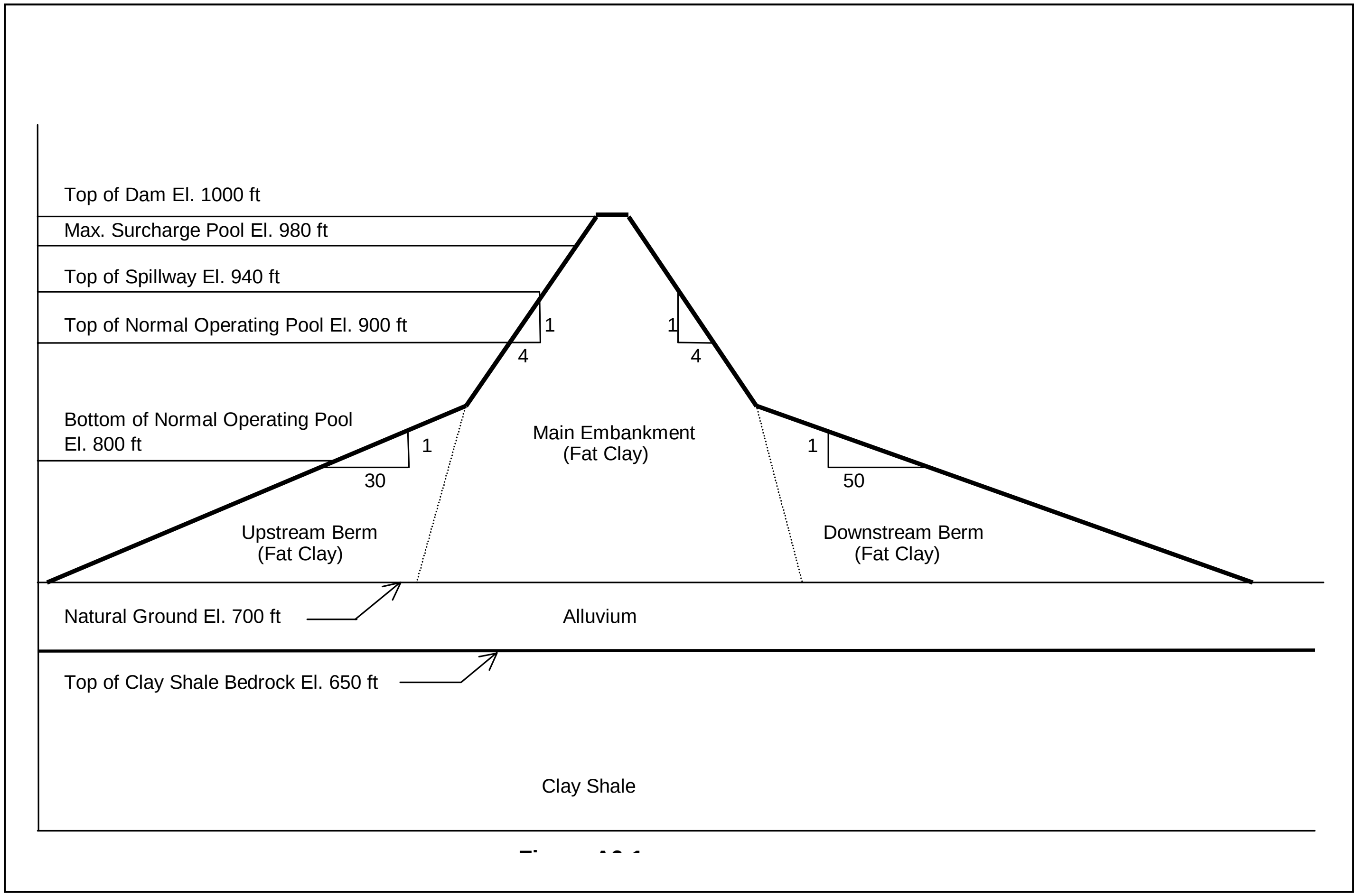
EXAMPLE PROBLEM FOR STEADY STATE SEEPAGE, SLOPE STABILITY RELIABILITY ANALYSIS

I-1. Introduction. Construction on Dam X started in 1950 and was completed in 1960. The project is a multi-purpose project for flood control, hydropower, environmental restoration and recreation. The dam is a rolled earth embankment and is approximately 300 feet high and approximately 2 miles long. The material used for the embankment came from required excavations and consists of fat clays and is fairly homogeneous, see Figure I-1. Dam X is a high hazard dam with several large cities located downstream within the flood plain.

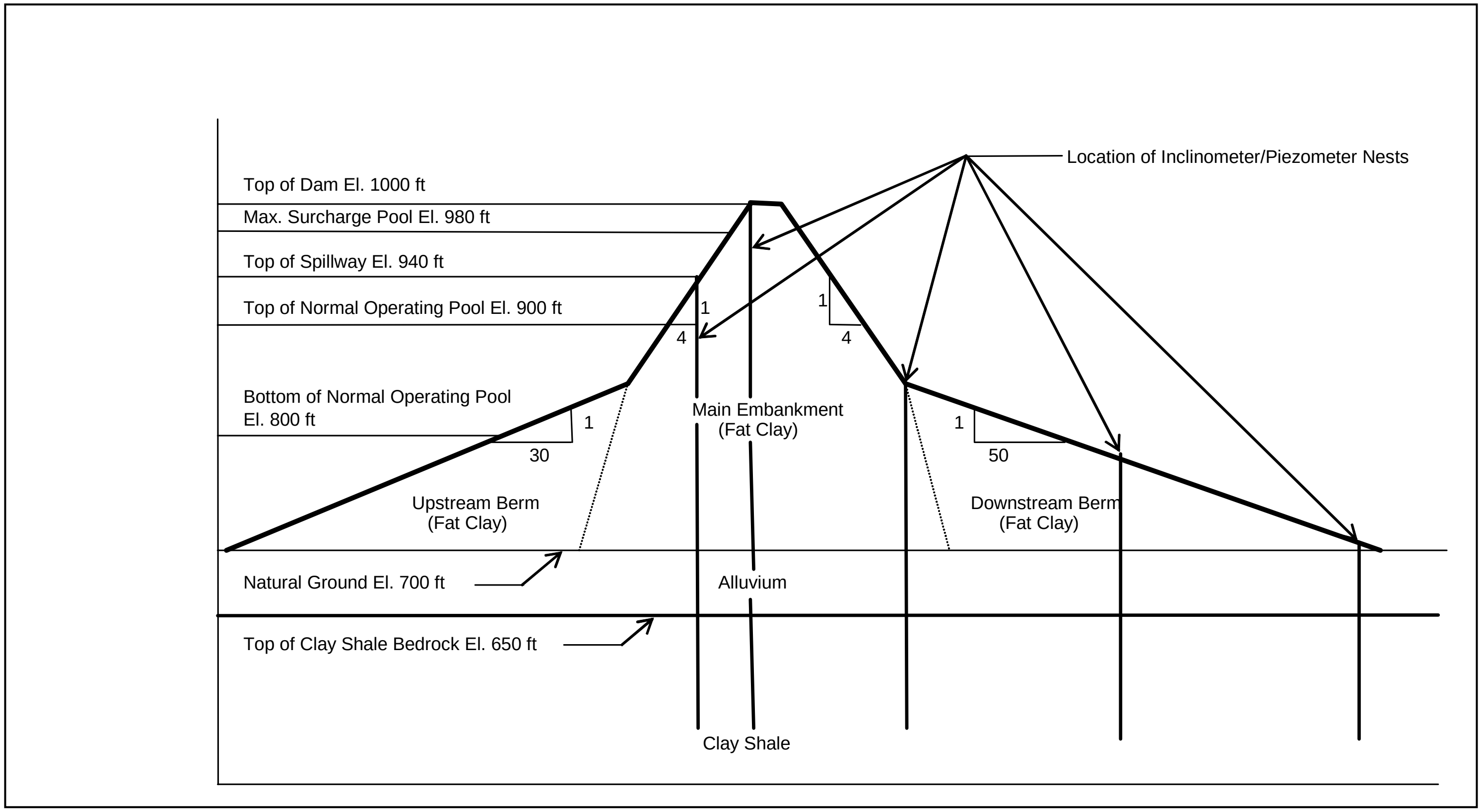
I-2. Current Project Conditions.

a. The original designers were concerned with the foundation conditions in an area approximately 1,000 feet long near the left abutment. The concern centered around a calcarious marine clay shale bedrock formation which is highly fractured and indicative of a high degree of movement during the river valley development. The original designers were concerned about how the foundation would behave after it was loaded with the embankment. Therefore, this area of the project was highly instrumented with settlement gauges, inclinometers and piezometers to monitor movement both during and after construction. Piezometers and inclinometers were placed in nests approximately 250 feet upstream of the crest of the embankment, at the upstream edge of the crest of the embankment and approximately 500 feet, 1000 feet and 1,500 feet downstream of the crest of the embankment, see Figure I-2. Settlement gauges were installed in the crest of the embankment. The normal operating pool ranges from elevation 800 feet to 900 feet. The exclusive flood control zone is between elevation 900 feet and 940 feet. The maximum surcharge pool or the maximum design pool elevation is 980 feet. Except for the historic record pool, the pool has fluctuated between elevation 800 and 900 feet. Some of the inclinometer data show movement in the downstream direction during and after construction within the clay shale bedrock. The data shows the rate of movement, in general, slowing down with time.

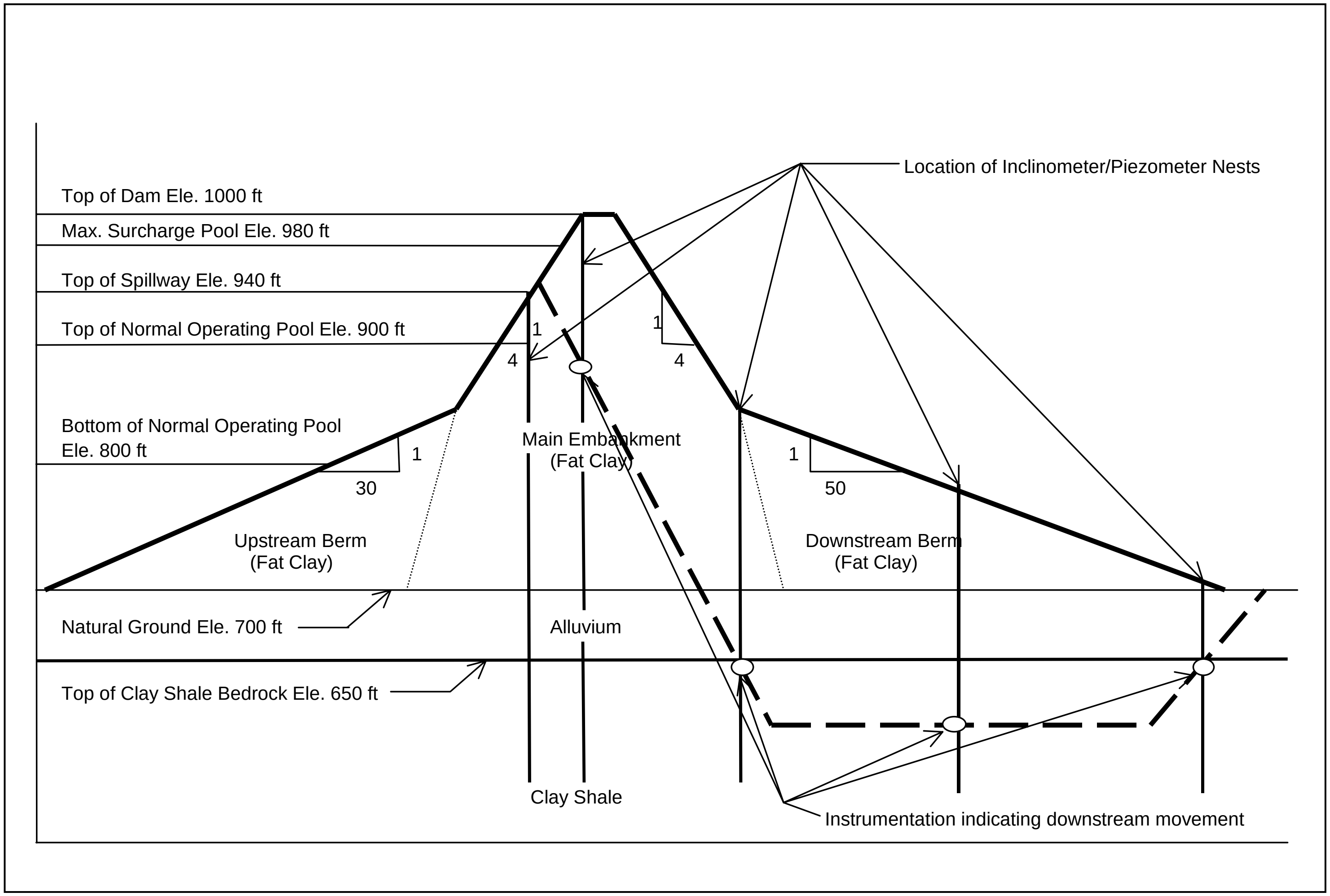
b. A recent flood event caused the pool to reach its record pool elevation of 940 feet (approximately 40 feet above the previous record pool). Shortly after the flood event, a physical inspection of the embankment showed signs of active movement. Cracks along the crest of the embankment where identified within the 1,000 foot reach describe above. The cracks also exhibited a vertical offset of approximately ½ of an inch when measured from the upstream side of the crack to the downstream side of the crack. A review of the instrumentation data revealed that the movements appeared to accelerate in response to the high reservoir pool. This situation is a significant concern since the maximum design pool is approximately 40 feet higher than the historic record pool. The review of instrumentation data also revealed that the movement appeared to represent a classic slope stability wedge failure, see Figure I-3.



Cross Section of Dam X
Figure I-1



Instrumentation Locations
Figure I-2



Movement Surface
Figure I-3

I-3. Problem Definition.

Additional sampling and testing of the bedrock shale material, within the movement zone, was performed after the signs of movement were identified. The testing revealed substantially lower shear strengths than those used in the slope stability analysis that was conducted in the original design phase of the project. The geologic history suggests that because of the movement experienced during valley formation, that the residual shear strength of the bedrock material had been mobilized and the new test results showed the shale material to be at residual shear strength. After reviewing the historic piezometeric data and comparing that to the assumptions made in the original design analysis, the design assumptions appear to be reasonable.

I-4. Project Site Characterization.

Table I-1 lists the representative soil properties used for the original design stability analysis. Table I-1 also gives the expected values for the same soil properties. In general, the expected values are different than the values used for design purposes. Within the Corps of Engineers, the 1/3 2/3 rule (the strength envelope used for design is drawn generally such that 1/3 of the test values are below the envelope and 2/3 of the test values are above) is generally used. The expected value is the value the designer thinks the material will exhibit in the field and can be substantially different than the values used for design purposes. The expected value for the weak shale is based on sampling and testing (that was conducted after the pool of record flood event) of the material located in movement zone.

Table I-1. Material Properties

Material Type	Unit Weight		Shear Strength Cohesion Φ			
	Design Value	Expected values	Design Value		Expected values	
			C	Φ	C	Φ
Embankment	115 pcf	125 pcf	0	23.0^o	0	27.1^o
Alluvial	100 pcf	105 pcf	0	15.0^o	0	18.0^o
Firm Shale	120 pcf	128 pcf	0	17.0^o	0	20.8^o *
Weak Shale	NA	102 pcf	NA	NA	0	8.0^{o*} *

* This strength represents the cross bed shear strength in the shale.
** Residual shear strength of the shale.

I-5. Modes of Failure.

a. Looking at the physical distress and movement data from the inclinometers it appears that the problems experienced at Dam X are slope stability related. Therefore, a re-analysis of the stability of the dam was conducted at pool elevation 940 and 980 feet, in order to verify the conclusions of the instrumentation data. Using the expected values and the new shear strength data for the shale bedrock material along the apparent movement zone, a stability analysis was performed for a pool elevation of 940 and 980 feet.

b. The results of the analysis are shown below in Table I-2.

Table I-2. Results of Static Stability Analysis

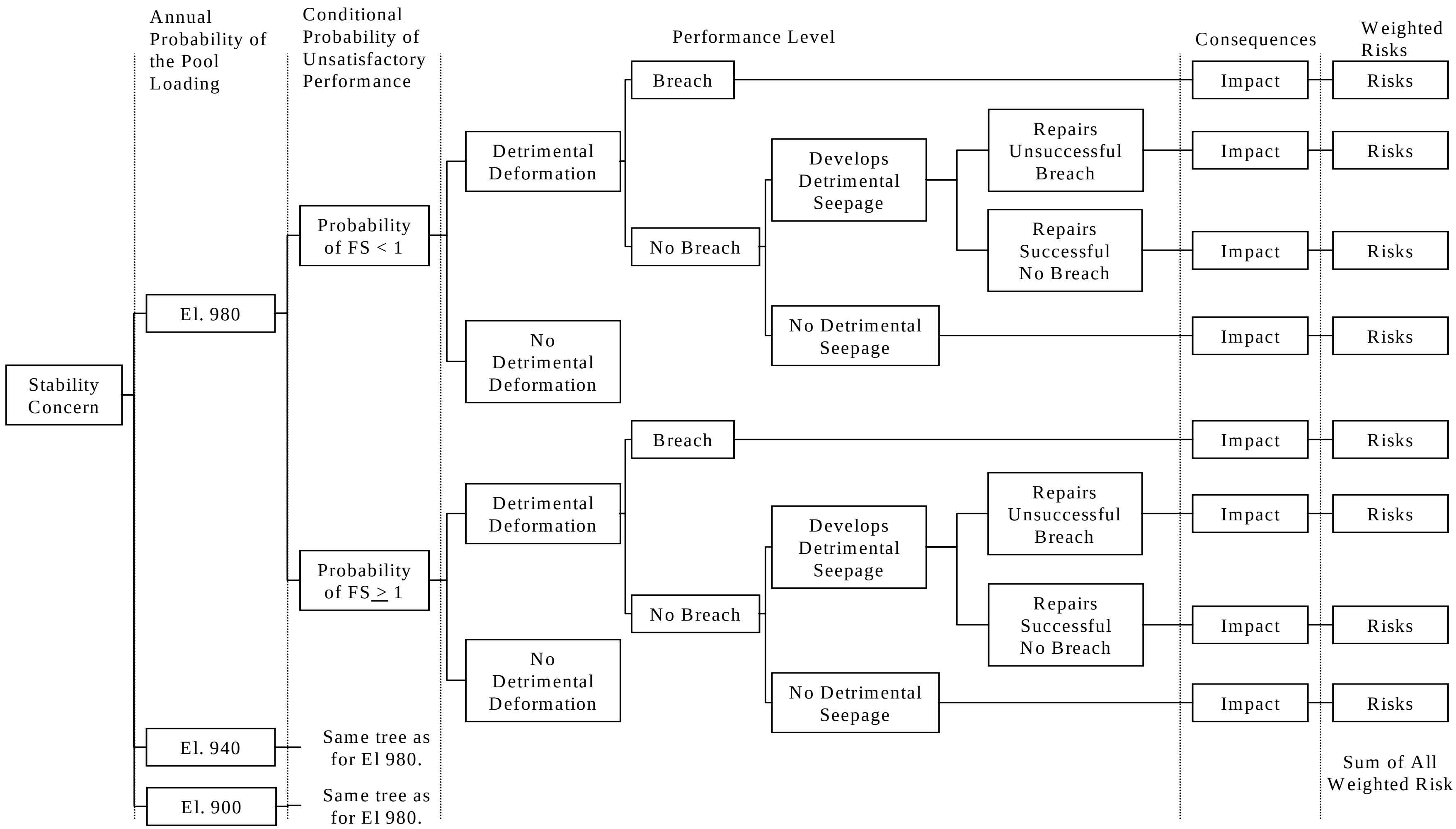
Reservoir Elevation	Factor of Safety	
	Original Design	Re-Analysis
940 feet msl	1.500	1.100
980 feet msl	1.200	0.997

c. The re-analysis shows that there is a significant concern associated with the stability of Dam X and a Major Rehabilitation Evaluation Report is warranted. The first thing that must be done after a decision is made to conduct a Major Rehabilitation Evaluation Report is to develop an event tree.

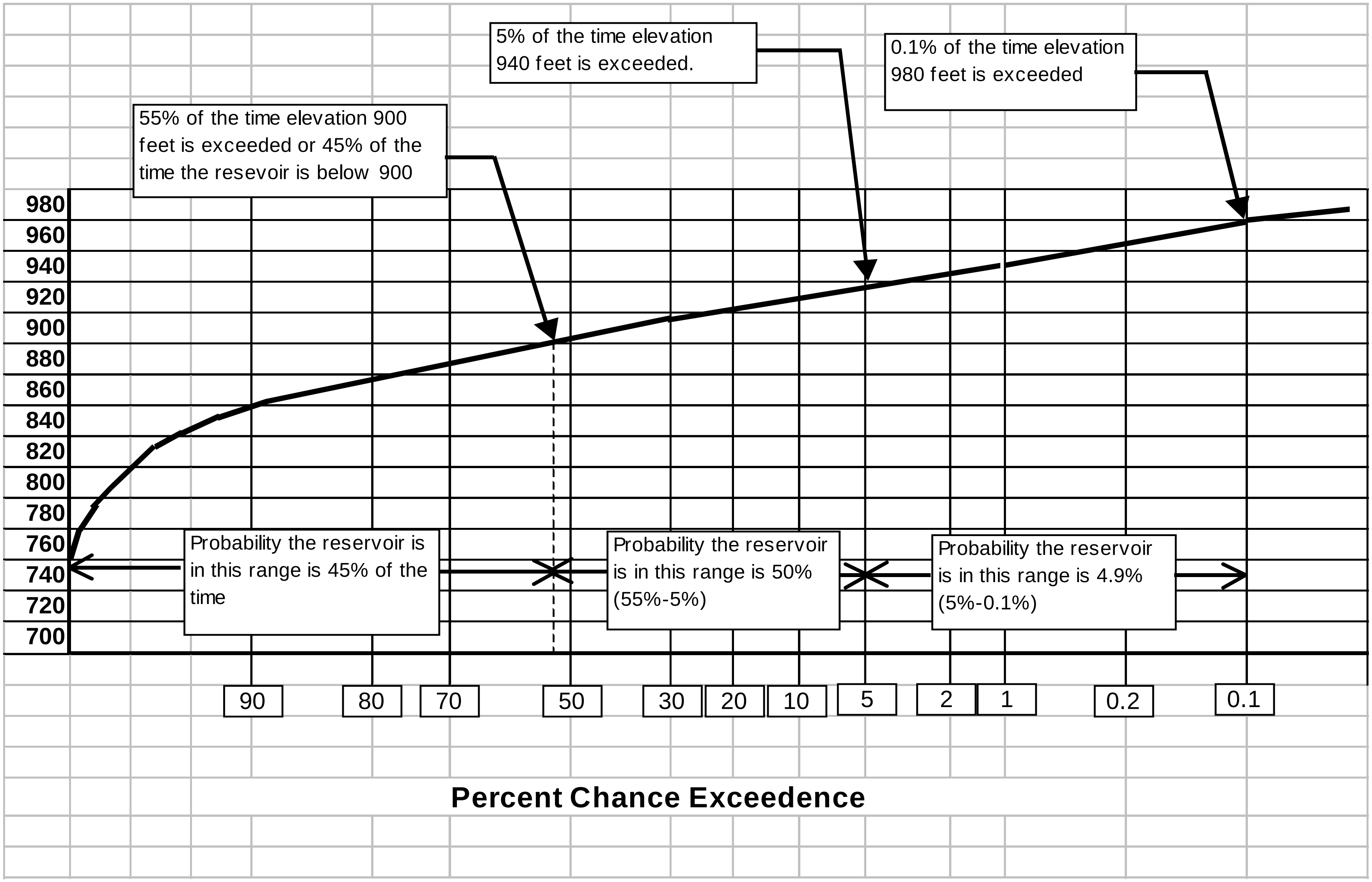
I-6. Event Tree.

a. An event tree is used to describe the possible events and the outcomes of those events given the current conditions of the project. For this example, an event tree was developed and is shown in Figure I-4. The event tree contains five major categories. The categories consist of events that are related to the frequency of loading (in this example the probability of the pool being at various elevations), the conditional probability of unsatisfactory performance, the performance level, consequences, and weighted damages (risk), that could occur for a particular set of events. In this case to determine the frequency of loading, a pool probability curve is needed, see Figure I-5. To select the number of cases to examine, the threshold condition must be determined. The threshold condition is simply the pool elevation at which below this condition there is not a problem and above this level problems start to occur. For Dam X the threshold elevation was selected at reservoir elevation 900 feet. This elevation was selected because there are no historic problems in either the instrumentation data or physical conditions in response to a pool elevation of 900 feet or below. The reservoir elevation has been there several times in the past and for extended periods of time with no adverse conditions identified.

b. Three reservoir pool elevations were selected for the event tree. Elevation 900 feet (threshold value), Elevation 940 feet (elevation at which problems were identified) and Elevation 980 feet (which is the maximum surcharge pool). The probability of loading for these elevations are calculated by using Figure I-5.



Event Tree
Figure I-4



Pool Probability Curve for Dam X
Figure I-5

I-7. Conditional Probability of Unsatisfactory Performance. For the purpose of this example problem, it is assumed that a factor of safety of 1.100 is considered unsatisfactory performance because of the physical movement that occurred at a pool elevation of 940 feet. To determine the conditional probability of unsatisfactory performance, the Taylor Series Method was used as described in Appendix D and ETL 1110-2-547.

I-8. Steps in the Reliability Analysis.

Step 1 - Determine the expected value (EV) of all the input parameters that are used to determine the factor of safety, see Table I-3.

Step 2 - Estimate the standard deviation of the input parameters. For the purpose of this example problem the standard deviation for the input parameters are contained in Table I-3.

Step 3 – Compute the factor of safety using the expected values for each parameter. Compute the factor of safety for each parameter increased and decreased by one standard deviation from the expected value while holding all others at the expected value, for each of the pool loading conditions, see Tables I-4 through I-6.

Step 4 - Compute the change in the factor of safety for each parameter. This is done by simply subtracting the factor of safety for the EV + one standard deviation from the factor of safety for the EV – one standard deviation, see Tables I-4 through I-6.

Table I-3. Expected Values and Standard Deviation of the Input Parameters

Variable	Expected Value	Standard Deviation
Shear Strength		
Embankment	27.1^{o}	2.56^{o}
Alluvium	18.0^{o}	3.92^{o}
Firm Shale	20.8^{o}	5.57o
Weak Shale	8.0^{o}	3.5^{o}
Unit Weight		
Embankment	125 pcf	5.67 pcf
Alluvium	105 pcf	2.33 pcf
Firm Shale	128 pcf	4.33 pcf
Weak Shale	102 pcf	6.67 pcf

Table I-4. Reliability Calculation for Pool Elevation 900 feet

FACTOR OF SAFETY USING VARIABLES ASSIGNED THEIR EXPECTED VALUES = 1.500					
VARIABLE	Expected VALUE	ONE STANDARD DEVIATION	PLUS/MINUS ONE STANDARD DEVIATION	F.S. PLUS/MINUS ONE STANDARD DEVIATION	DELTA F.S.
Shear Strength					
			29.66°	1.525	
Embankment	27.1°	2.56°			0.057
			24.54°	1.468	
			21.92°	1.515	
Alluvium	18.0°	3.92°			0.083
			14.08°	1.432	
			26.37°	1.496	
Firm Shale	20.8°	5.57°			0.008
			15.23°	1.488	
			11.50°	1.492	
Weak Shale	8.0°	3.50°			0.193
			4.50°	1.299	
Unit Weight					
			130.67	1.376	
Embankment	125	5.67			-0.043
			119.33	1.419	
			107.33	1.510	
Alluvium	105	2.33			0.028
			102.67	1.482	
			132.33	1.500	
Firm Shale	128	4.33			0.007
			123.67	1.493	
			108.67	1.502	
Weak Shale	102	6.67			0.011
			95.33	1.491	

Table I-5. Reliability Calculation for Pool Elevation 940 feet

FACTOR OF SAFETY USING VARIABLES ASSIGNED THEIR EXPECTED VALUES = 1.100

VARIABLE	Expected VALUE	ONE STANDARD DEVIATION	PLUS/MINUS ONE STANDARD DEVIATION	F.S. PLUS/MINUS ONE STANDARD DEVIATION	DELTA F.S.
Shear Strength					
			29.66°	1.127	
Embankment	27.1°	2.56°			0.063
			24.54°	1.064	
			21.92°	1.115	
Alluvium	18.0°	3.92°			0.080
			14.08°	1.035	
			26.37°	1.096	
Firm Shale	20.8°	5.57°			0.008
			15.23°	1.088	
			11.50°	1.192	
Weak Shale	8.0°	3.50°			0.190
			4.50°	1.002	
Unit Weight					
			130.67	1.066	
Embankment	125	5.67			-0.063
			119.33	1.129	
			107.33	1.110	
Alluvium	105	2.33			0.101
			102.67	1.009	
			132.33	1.100	
Firm Shale	128	4.33			0.005
			123.67	1.095	
			108.67	1.105	
Weak Shale	102	6.67			0.010
			95.33	1.095	

Table I-6. Reliability Calculation for Pool Elevation 980 feet

FACTOR OF SAFETY USING VARIABLES ASSIGNED THEIR EXPECTED VALUES = .997					
VARIABLE	Expected VALUE	ONE STANDARD DEVIATION	PLUS/MINUS ONE STANDARD DEVIATION	F.S. PLUS/MINUS ONE STANDARD DEVIATION	DELTA F.S.
Shear Strength					
			29.66^o	1.051	
Embankment	27.1^o	2.56^o			0.060
			24.54^o	0.991	
			21.92^o	1.061	
Alluvium	18.0^o	3.92^o			0.078
			14.08^o	0.983	
			26.37^o	1.001	
Firm Shale	20.8^o	5.57^o			0.006
			15.23^o	0.995	
			11.50^o	0.998	
Weak Shale	8.0^o	3.50^o			0.191
			4.50^o	0.807	
Unit Weight					
			130.67	1.021	
Embankment	125	5.67			-0.060
			119.33	1.081	
			107.33	1.042	
Alluvium	105	2.33			0.101
			102.67	0.941	
			132.33	1.031	
Firm Shale	128	4.33			0.006
			123.67	1.025	
			108.67	1.011	
Weak Shale	102	6.67			0.011
			95.33	1.000	

Step 5 - Compute the standard deviation of the factor of safety. The formula for the standard deviation of the factor of safety is computed using the following equation.

$$\sigma_F = \sqrt{(\Delta F_1/2)^2 + (\Delta F_2/2)^2 + (\Delta F_3/2)^2 + \ldots}$$

For this example problem, $\sigma_{F\text{-}900}$ is calculated as follows:

$$\sigma_{F\text{-}900} = \sqrt{(0.057/2)^2 + (0.083/2)^2 + (0.008/2)^2 + (0.193/2)^2 + (-0.043/2)^2 + (0.028/2)^2 + (0.007/2)^2 + (0.011/2)^2}$$

$$\sigma_{F\text{-}900} = 0.1121$$

For this example problem, $\sigma_{F\text{-}940}$ is calculated as follows:

$$\sigma_{F\text{-}940} = \sqrt{(0.063/2)^2 + (0.080/2)^2 + (0.008/2)^2 + (0.190/2)^2 + (-0.063/2)^2 + (0.101/2)^2 + (0.005/2)^2 + 0.010/2)^2}$$

$\sigma_{F\text{-}940} = 0.1233$

For this example problem, $\sigma_{F\text{-}980}$ is calculated as follows:

$$\sigma_{F\text{-}980} = \sqrt{(0.060/2)^2 + (0.078/2)^2 + (0.006/2)^2 + (0.191/2)^2 + (-0.060/2)^2 + (0.101/2)^2 + (0.006/2)^2 + (0.011/2)^2}$$

$\sigma_{F\text{-}980} = 0.1226$

Step 6 **-** Compute the Coefficient of Variation. The formula to compute the Coefficient of Variation (COV) is as follows:

$$COV = \sigma_F/F_{EV}$$

Where F_{EV} is the factor of safety computed using the expected values for all the input parameters.

For this example problem, the COV is:

$$COV_{900} = 0.1121/1.5 \text{ x } 100\% = 7.47\%$$

$$COV_{940} = 0.1233/1.1 \text{ x } 100\% = 11.21\%$$

$$COV_{980} = 0.1226/0.997 \text{ x } 100\% = 12.27\%$$

Step 7 **-** Compute the probability that the factor of safety is less than one.

Using the value of F_{EV} and the value of COV, compute the value of the log normal reliability index (β_{LN}) as follows:

$$\beta_{LN} = LN[F_{EV}/ (1+(COV)^2)^{1/2}] / [LN (1 + (COV)^2)]^{1/2}$$

Using Table D-1 of Appendix D or other published statistical tables find the standard cumulative normal distribution function of $\beta_{LN.}$ This value is the reliability. The probability of failure (P_f) is 1 minus this value.

For Example:
If $\beta_{LN} = 2.32$, then from Table D-1, the normal distribution function of 2.32 equals 0.9898 (the reliability). The probability of failure (P_f) is 1-0.9898 = 0.0102.

Using the mathematical procedure presented above gives the following results for this example problem:

A probability of the factor of safety is less than 1 for a pool elevation of 900 feet is approximately 0%. This tells us that at a reservoir elevation of 900 feet we are fairly confident that the dam is stable at elevation 900 feet.

A probability of the factor of safety is less than 1 for a pool elevation of 940 feet is approximately 21%.

A probability of the factor of safety is less than 1 for a pool elevation of 980 feet is approximately 50%.

These numbers are then added to the event tree. For example at elevation 980 feet, the event entitled “Probability of a FS < 1” would be assigned a 50% probability based on the calculations performed above.

I-9. Performance Level. The performance levels are developed as part of the event tree. These events are simply the events that the team feels are likely out comes given unsatisfactory performance. A probability of occurrence is assigned to each of these events. If an analytical tool is not available to compute the probability of occurrence, then expert elicitation is one method to perform this task. In the case of Dam X, the performance levels will be determined by use of expert elicitation and engineering judgement. These probabilities are then added to the event tree as well. Once all the probabilities have been assigned to each event, the probabilities are multiplied across the branch and then multiplied by the consequences to determine the annual weighted damages (risk).